

# Supersymmetric $N = 2$ Einstein-Yang-Mills Monopoles and Covariant Attractors

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## Abstract

We present two generic classes of supersymmetric solutions of  $N = 2, d = 4$  supergravity coupled to non-Abelian vector supermultiplets with a gauge group that includes an  $SU(2)$  factor. The first class consists of embeddings of the 't Hooft-Polyakov monopole and in the examples considered it has a fully regular, asymptotically flat space-time metric without event horizons. The other class of solutions consists of regular non-Abelian extreme black holes. There is a *covariant attractor* at the horizon of these non-Abelian black holes.

## Introduction

The search for and study of supersymmetric supergravity solutions having the interpretation of long-range fields of string states, has been one of the most fruitful fields of theoretical research for the last fifteen years. In 4-dimensional theories (in particular, in  $N = 2$  supergravities), most of the effort has been directed to find and study black holes with Abelian charges in flat spacetime. The most general black-hole-type solutions of these theories (ungauged  $N = 2, d = 4$  supergravity coupled to vector supermultiplets) were found

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in Ref. [1]<sup>5</sup>. This, and the existence of the attractor mechanism [6] and its relations to stringy black hole entropy calculations or to topological strings are two of the main results obtained so far.

These results have not been extended to black holes with non-Abelian charges. Actually, the little work that has been done so far in supergravity theories with non-Abelian Yang-Mills fields concerns magnetic monopoles and not black holes. Two main results in this direction have been the construction of two supersymmetric gravitating monopole solutions in  $N = 4, d = 4$  theories by Harvey and Liu [4] and Chamseddine and Volkov [5] whose metrics have neither singularities nor event horizons. They have not been related to black holes and, to the best of our knowledge there is no microscopic interpretation of these massive, charged, objects that are not black holes but may be elementary constituents of them. On the other hand, apart from those which are just embeddings of Abelian solutions into non-Abelian theories, not many black-hole solutions of these theories are known in analytic form [7] and without an analytical form it is very difficult to address questions about the existence of attractors in black holes with non-Abelian charges.

Our aim in this article is to start filling this gap in our knowledge of supersymmetric supergravity solutions with non-Abelian Yang-Mills fields, studying, in particular, black-hole and monopole-type solutions. We are going to present an extension of the results of [2, 8], characterizing the most general static supersymmetric solutions in  $N = 2, d = 4$  supergravity coupled to non-Abelian vector supermultiplets<sup>6</sup>, to which we shall refer as  $N = 2, d = 4$  Einstein-Yang-Mills theory. In this theory only the isometries of the special-Kähler manifold parametrized by the scalars in the vector multiplets are gauged, which leads to a positive-semidefinite scalar potential. This characterization simplifies the search for supersymmetric black-hole solutions and we are going to use it to study the construction of solutions in models that admit an  $SO(3)$  gauge group. We are going to present some complete analytic solutions for two models addressing the questions concerning attractors raised above. We are going to see that both models (and presumably all models, including the *stringy* ones) admit solutions in which the Yang-Mills fields describe an 't Hooft-Polyakov monopole and whose, asymptotically flat, metrics are completely regular and have no event horizons, just as it happens in the Harvey-Liu and Chamseddine-Volkov solutions. We will also show that these models (and, again, probably all other models) admit solutions with non-Abelian Yang-Mills fields with the same asymptotic behaviour as the 't Hooft-Polyakov monopoles, whose metrics are regular outside an event horizon. We will also describe how the attractor mechanism works in these examples.

Monopoles in  $N = 2$  gauge theories were first studied by D'Adda *et al.* in Ref. [10], and one of the models we are going to study is probably its closest supergravity analogue:  $SO(3)$  gauged model on  $\overline{\mathbb{CP}}^3$ . In fact, one can see that the rigid limit of the model, see *e.g.* [11], explicitly leads to the theory studied in [10].  $SO(3)$  monopoles in EYM were

<sup>5</sup>The proof that they are the most general solutions of that kind was given in Ref. [2], where all the supersymmetric solutions of these theories were found. In presence of  $R^2$  corrections it was shown in Ref. [3] that they have the same form as those in Ref. [1].

<sup>6</sup>The general problem will be considered in Ref. [9].

also studied in Ref. [12], but the model used there is by itself not a supergravity theory as their scalar manifold is not special Kähler. For a specific value of the dilaton coupling [13], however, the model corresponds to a truncation of a supergravity and the monopoles correspond to the one found by Harvey and Liu [4]. The second model that we shall consider closely resembles Harvey and Liu's set up and is the  $\mathcal{ST}[2, n]$  model.

The plan of this article is as follows: in Section 1 we will review  $N = 2 d = 4$  supergravity coupled to non-Abelian vector supermultiplets and describe the characterization of the supersymmetric solutions in the timelike class, thus obtaining the minimal set of equations that need to be solved in order to have supersymmetric solutions. In Section 2, we will discuss how one can construct solutions for generic models with an  $SO(3)$  gauge group and on Sections 3 and 4 we will consider two specific examples of  $N = 2$  theory with that gauge group and we will study the complete solutions constructed with the above methods. Finally, in Section 5 we will discuss our results and present our conclusions and future directions of research, some of which are under investigation.

## 1 $N = 2, d = 4$ EYM supergravity

We start by describing the theory of  $N = 2 d = 4$  supergravity coupled to non-Abelian vector supermultiplets to which we will refer to as  $N = 2$  Einstein-Yang-Mills (EYM). These theories can be obtained from the ungauged theory with vector supermultiplets by gauging the isometries of the special-Kähler manifold parametrized by the scalars in the vector supermultiplets<sup>7</sup>. We, however, do not consider the most general gaugings, but rather restrict ourselves to gaugings that act block-diagonally on the symplectic sections defining the theory. Another way of stating this property is that, in case a prepotential exists, we are going to gauge symmetries of the prepotential. Hence, we are going to gauge groups  $G \subseteq Sl(\bar{n}, \mathbb{R})$ , where  $n = \bar{n} - 1$  is the number of vector multiplets.

The bosonic part of the action for these theories is given by

$$\begin{aligned} S = & \int d^4x \sqrt{|g|} [R + 2\mathcal{G}_{ij*}\mathfrak{D}_\mu Z^i \mathfrak{D}^\mu Z^{*j} + 2\text{Sym} \mathcal{N}_{\Lambda\Sigma} F^{\Lambda\mu\nu} F^\Sigma{}_{\mu\nu} \\ & - 2\Re e \mathcal{N}_{\Lambda\Sigma} F^{\Lambda\mu\nu*} F^\Sigma{}_{\mu\nu} - V(Z, Z^*)] , \end{aligned} \quad (1.1)$$

and, for vanishing fermions, the supersymmetry transformation rules of the fermions are

$$\delta_\epsilon \psi_{I\mu} = \mathfrak{D}_\mu \epsilon_I + \epsilon_{IJ} T^+{}_{\mu\nu} \gamma^\nu \epsilon^J , \quad (1.2)$$

$$\delta_\epsilon \lambda^{Ii} = i \not{\partial} Z^i \epsilon^I + \epsilon^{IJ} [\mathcal{G}^{i+} + W^i] \epsilon_J . \quad (1.3)$$

The supersymmetry transformation laws for the bosons are the same as in the ungauged case [2]. This will have important consequences in what follows.

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<sup>7</sup>For a more detailed description see Refs. [9] or [14], the review Ref. [15], and the original works Refs. [16, 17]. Our conventions are contained in Refs. [2, 8].

In the above equations

$$\mathfrak{D}_\mu Z^i = \partial_\mu Z^i + g A^\Lambda_\mu k_\Lambda{}^i, \quad (1.4)$$

where  $k_\Lambda{}^i(Z)$  are the holomorphic Killing vectors of the metric  $\mathcal{G}_{ij*}$ , is the gauge covariant derivative acting on the scalars; the covariant derivative acting on the Killing spinor is

$$\mathfrak{D}_\mu \epsilon_I = \left\{ \nabla_\mu + \frac{i}{2} \mathcal{Q}_\mu + \frac{i}{2} g A^\Lambda_\mu \mathcal{P}_\Lambda \right\} \epsilon_I, \quad (1.5)$$

where  $\mathcal{Q}_\mu$  is the pullback of the Kähler 1-form and  $\mathcal{P}_\Lambda$  is the momentum map satisfying

$$k_{\Lambda i*} = i \partial_{i*} \mathcal{P}_\Lambda, \quad (1.6)$$

is the Lorentz- Kähler- and gauge-covariant derivative acting on spinors and

$$W^i = \frac{1}{2} g \mathcal{L}^{*\Lambda} k_\Lambda{}^i. \quad (1.7)$$

The potential  $V(Z, Z^*)$ , is given by

$$V(Z, Z^*) = 2 \mathcal{G}_{ij*} W^i W^{j*} = -\frac{1}{4} g^2 (\Im \mathcal{M})^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma. \quad (1.8)$$

The negative-definiteness of  $\Im \mathcal{M}_{\Lambda\Sigma}$  and the reality of the momentum map imply that  $V \geq 0$ .

We are interested in supersymmetric solutions of the above system admitting at least one Killing spinor  $\epsilon_I$ . Their general form can be found following Refs. [2, 8] and our discussion will be extremely brief. As usual, if  $\epsilon_I$  is a Killing spinor, the bilinear  $V^\mu = i \bar{\epsilon}^I \gamma^\mu \epsilon_I$  is a non-spacelike Killing vector. We consider only the case in which it is timelike, i.e.  $V^2 = 4|X|^2 > 0$ , and introduce a time coordinate  $t$  by  $V^\mu \partial_\mu = \sqrt{2} \partial_t$ .

From the gaugino variation (1.3) we get the equation

$$V^\mu \mathfrak{D}_\mu Z^i + 2X W^i = 0, \quad (1.9)$$

whose analogue in the ungauged case states that the scalars  $Z^i$  are time-independent. In the gauged case, we can obtain time-independence by choosing the gauge fixing

$$A^\Lambda{}_t = -\sqrt{2}|X|^2 \mathcal{R}^\Lambda, \quad (1.10)$$

which solves Eq. (1.9) due to the property  $\mathcal{L}^\Lambda k_\Lambda^i = 0$  [14].

The other three vectors  $V^m$  ( $m = 1, 2, 3$ ) that we can construct as bilinears of the Killing spinor are exact 1-forms, and can thence be used to define spatial coordinates  $x^m$  by  $V^m \equiv dx^m$ . The metric is of the conformastationary form

$$ds^2 = 2|X|^2(dt + \omega)^2 - \frac{1}{2|X|^2} dx^m dx^m, \quad (1.11)$$

where  $\omega = \omega_m dx^m$  is a possible 1-form. As we are interested in static spacetimes we are going to take  $\omega = 0$ . This choice imposes a constraint which can be written as

$$\langle \mathcal{I} | \mathfrak{D}_m \mathcal{I} \rangle = 0 . \quad (1.12)$$

where we have used the variables that will govern the solutions:

$$\mathcal{R} \equiv \Re(\mathcal{V}/X) , \mathcal{I} \equiv \Im(\mathcal{V}/X) \longrightarrow \frac{1}{2|X|^2} = \langle \mathcal{R} | \mathcal{I} \rangle , \quad (1.13)$$

Observe that up to the replacement of the ordinary derivative by the gauge-covariant derivative, the constraint has the same functional form as in the Abelian case.

From the gravitino and gaugino variations we deduce the symplectic vector of 2-form field-strengths:

$$F = -\sqrt{2} \mathfrak{D}(|X|^2 \mathcal{R} dt) - \sqrt{2} |X|^2 \star (dt \wedge \mathfrak{D}\mathcal{I}) , \quad (1.14)$$

which, again, has the same functional form as in the ungauged case and is, moreover, consistent with the gauge fixing (1.10)!

Following the steps outlined in [2, Sec. 4.3] one can readily check that all the configurations of the above form are, at least, 1/2-BPS.

We still have to impose the equations of motion in order to find supersymmetric solutions. As any other symmetry of an action functional, supersymmetry implies relations between equations of motion. In contrast to other symmetries, however, supersymmetry implies relations between the e.o.m. of fields of different spin. This opens up the possibility to find a minimal set of e.o.m.s that need to be solved explicitly as to insure that all e.o.m.s are solved.

The most economical way of finding such a minimal set is by means of the Killing Spinor Identities (KSIs) [18, 19], which are off-shell relations between the equations of motion of the bosons of a supersymmetric theory. A remarkable characteristic of the KSIs is that their functional form depends only on the structure of the supersymmetry transformation rules of the bosons. As we remarked above, in the case of  $N = 2$  EYM the supersymmetry transformations of the bosons are exactly the same as in the ungauged case: this implies that the KSIs have the same form as in the ungauged case (given in Ref. [2, Sec. 3.1.1]) even if the equations of motion are different. Seeing this, we must conclude that in order to be sure that the configuration we obtained above solves the equations of motions, we only need to impose the Bianchi identities and the Yang-Mills equation.

The Bianchi identities for the supersymmetric field strengths Eq. (1.14) take the form

$$\mathfrak{D}_m \mathfrak{D}_m \mathcal{I}^\Lambda = 0 , \quad (1.15)$$

and the YM equations take the form

$$\mathfrak{D}_m \mathfrak{D}_m \mathcal{I}_\Lambda = \tfrac{1}{2} g^2 [f_{\Lambda(\Sigma}{}^\Gamma f_{\Delta)\Gamma}{}^\Omega \mathcal{I}^\Sigma \mathcal{I}^\Delta] \mathcal{I}_\Omega . \quad (1.16)$$

Eqs. (1.15) and (1.16) form a complicated system to solve, but not as complicated as one might have anticipated: in principle one might have imagined the appearance of  $\mathcal{R}$  in

the system which, seeing that they are functions of  $\mathcal{I}$ , would make the solutions highly non-linear. Fortunately this does not happen and we end up with a nice hierarchical picture: first solve (1.15) as to obtain the pair  $(A^\Lambda, \mathcal{I}^\Lambda)$  and use this information to find a solution to Eq. (1.16). Then solve the stabilization equations to obtain  $\mathcal{R}$  and use this to calculate  $|X|^2$  through Eq. (1.13). Lastly, as we imposed staticity, we must check Eq. (1.12) and its integrability equation

$$\langle \mathcal{I} | \mathfrak{D}_m \mathfrak{D}_m \mathcal{I} \rangle = 0 , \quad (1.17)$$

in order to avoid singularities like the ones studied in Refs. [20, 21].

Eq. (1.15) is, of course, a hard nut to crack, and it is a better idea to start with a given  $A^\Lambda$  and try to distill an  $\mathcal{I}^\Lambda$  from it by comparing the resulting field strength with the expression (1.14). Doing so, we find Eq. (1.10) and

$$\frac{1}{2} \epsilon_{pmn} F_{mn}^\Lambda = -\frac{1}{\sqrt{2}} \mathfrak{D}_p \mathcal{I}^\Lambda . \quad (1.18)$$

This equation is readily recognised as the Bogomol'nyi equation [22] and allows us to embed YM solutions satisfying it (*e.g.* monopoles) into  $N = 2$  EYM theories. In the next section we are going to work out some of these solutions.

## 2 Solutions of $SO(3)$ $N = 2$ EYM

For brevity let us only consider  $N = 2$  EYM systems containing an  $SO(3)$  gauge group, parametrizing the directions in which the  $SO(3)$  acts with indices  $a = 1, 2, 3$  and ignoring for the moment the other directions. If we make the “hedgehog” Ansatz

$$\mathcal{I}^a = \mathcal{I}(r) n^a , \quad A^a{}_m = \Phi(r) \epsilon_{mn}{}^a n^n , \quad n^a \equiv x^a/r , \quad r \equiv \sqrt{x^b x^b} . \quad (2.1)$$

where  $\mathcal{I}$  and  $\Phi$  are functions of  $r$  alone, we see that the Bogomol'nyi equation (1.18) admits a 2-parameter ( $\mu$  and  $\rho$ ) family of solutions given by [23]

$$\begin{aligned} \mathcal{I}(r) &= \frac{\sqrt{2}\mu}{g} \mathsf{H}_\rho(\mu r) , \quad \mathsf{H}_\rho(r) = \coth(r + \rho) - \frac{1}{r} , \\ \Phi(r) &= \frac{\mu}{g} \mathsf{G}_\rho(\mu r) , \quad \mathsf{G}_\rho(r) = \frac{1}{r} - \frac{1}{\sinh(r + \rho)} . \end{aligned} \quad (2.2)$$

The next step is to obtain the  $\mathcal{I}_a$  from Eq. (1.16): a solution to this equation is readily found by observing that since  $\mathcal{I}_a$  has to be proportional to  $n^a$ , the r.h.s. of said equation vanishes identically. Also, the co-adjoint representation under which  $\mathcal{I}_a$  transforms, is the same as the adjoint representation, whence Eq. (1.16) reduces to Eq. (1.15). The result is

$$\mathcal{I}_a = \frac{g\mathcal{J}}{2} \mathcal{I}^a , \quad (2.3)$$

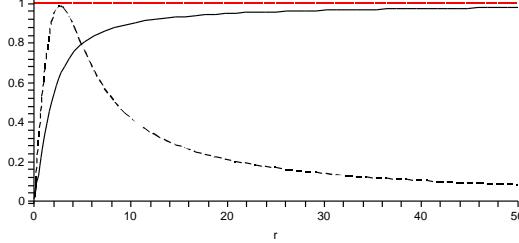


Figure 1: The profiles of the functions  $G_0$  and  $H_0$ .

where  $\mathcal{J}$  is an arbitrary constant.

The fact that  $\mathcal{I}_a$  has the same functional form as  $\mathcal{I}^a$  has consequences for the staticity condition Eq. (1.12): if we split the index  $\Lambda$  into an  $a$ -index and an  $u$ -index labelling the ungauged directions, we see that the condition (1.12) acts non-trivially only on the ungauged part, *i.e.*

$$\mathcal{I}_u d\mathcal{I}^u - \mathcal{I}^u d\mathcal{I}_u + \mathcal{I}_a \mathcal{D}\mathcal{I}^a - \mathcal{I}^a \mathcal{D}\mathcal{I}_a = \mathcal{I}_u d\mathcal{I}^u - \mathcal{I}^u d\mathcal{I}_u = 0, \quad (2.4)$$

which we can therefore solve as in the Abelian case.

At this point the solutions are completely determined. In order to find the explicit forms of  $\mathcal{R}$  and the spacetime metric, however, we must solve the stabilization equations which depend on the specific model under consideration. We will study two models that allow for an  $SO(3)$  gauging in Sections 3 and 4 and there we will discuss the physical properties of the complete solutions.

For now we are going to study two particularly interesting solutions of the above family: those with  $\rho = 0$  and those with  $\rho \rightarrow \infty$ .

## 2.1 $\rho = 0$ : 't Hooft-Polyakov Monopoles

The  $\rho = 0$  solution can be written, in our normalization, in the form

$$\begin{aligned} A_m^a &= \varepsilon_{mb}^a n^b \frac{\mu}{g} G_0(\mu r), & G_0(r) &= \frac{1}{r} - \frac{1}{\sinh r}, \\ \mathcal{I}^a &= \frac{\sqrt{2}\mu}{g} H_0(\mu r) n^a, & H_0(r) &= \coth r - \frac{1}{r}, \\ \mathcal{I}_a &= \frac{\mu\mathcal{J}}{\sqrt{2}} H_0(\mu r) n^a. \end{aligned} \quad (2.5)$$

The profile of the functions  $G_0$  and  $H_0$  are given Fig. (1). These functions are regular and bound between 0 and 1 and . Thus, we see that  $\mathcal{I}$  (whence also  $\mathcal{I}^a$  and  $\mathcal{I}_a$ ) are regular at  $r = 0$ . The YM fields of this solution are those of the 't Hooft-Polyakov monopole [24].

## 2.2 $\rho \rightarrow \infty$ : Black hedgehogs

In the limit  $\rho \rightarrow \infty$  the solution becomes

$$\begin{aligned} A^a_m &= \varepsilon_{mb}{}^a \frac{n^b}{gr}, \\ \mathcal{I}^a &= -\sqrt{2} \left( \mathcal{I}_\infty + \frac{1}{gr} \right) n^a, \quad \mathcal{I}_\infty \equiv -\frac{\mu}{g}, \\ \mathcal{I}_a &= -\frac{g\mathcal{J}}{\sqrt{2}} \left( \mathcal{I}_\infty + \frac{1}{gr} \right) n^a. \end{aligned} \tag{2.6}$$

These fields are singular at  $r = 0$ . This singularity makes the solution uninteresting in flat spacetime and this is, probably, the reason why it has not been considered before in the literature. However, the coupling to gravity may cover it by an event horizon in which case we would obtain a non-Abelian black hole solution which we call a “black hedgehog”.

## 3 Embedding in $\overline{\mathbb{CP}}^n$ models

As mentioned in Section 1, in order to construct models of gauged sugra one can start with the invariances of a prepotential  $\mathcal{F}$ . Some of the easiest models are those given by quadratic prepotentials and in the case of  $\overline{\mathbb{CP}}^n$  the prepotential reads<sup>8</sup>

$$\mathcal{F} = \frac{i}{4} \eta_{\Lambda\Sigma} \mathcal{X}^\Lambda \mathcal{X}^\Sigma, \quad \eta = \text{diag}(-, [+^n]). \tag{3.1}$$

The Kähler potential is straightforwardly derived to give

$$e^{-\mathcal{K}} = |\mathcal{X}^0|^2 - \sum_{i=1}^n |\mathcal{X}^i|^2 = 1 - \sum_{i=1}^n |Z^i|^2 \equiv 1 - |Z|^2, \tag{3.2}$$

resulting in the standard Fubini-Study metric on  $\overline{\mathbb{CP}}^n$

$$\mathcal{G}_{ij^*} = \frac{\delta_{ij^*}}{1 - |Z|^2} + \frac{Z^i Z^{*j^*}}{(1 - |Z|^2)^2}, \quad \mathcal{G}^{ij^*} = (1 - |Z|^2) \{ \delta^{ij^*} - Z^i Z^{*j^*} \}, \tag{3.3}$$

which is an Einstein metric, *i.e.*  $R(\mathcal{G}) = \bar{n} \mathcal{G}$ . In fact,  $\overline{\mathbb{CP}}^n$  can be identified with the symmetric space  $SU(1, n)/U(n)$ . Observe that Eq. (3.2) the coordinates  $Z^i$  are constrained by

$$0 \leq |Z|^2 < 1. \tag{3.4}$$

The stabilization equations can be readily solved in this model:

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<sup>8</sup>Even though we will fix  $n = 3$  in following subsections, for the moment we will leave  $n$  undetermined.

$$\mathcal{R}_\Lambda = -\frac{1}{2}\eta_{\Lambda\Sigma} \mathcal{I}^\Sigma , \quad \mathcal{R}^\Lambda = 2\eta^{\Lambda\Sigma} \mathcal{I}_\Sigma , \quad (3.5)$$

which allows us to write down the metrical factor in Eq. (1.13) in terms of the  $\mathcal{I}^\Lambda$  and  $\mathcal{I}_\Lambda$  as

$$-g_{rr} = \frac{1}{2|X|^2} = -\frac{1}{2} \mathcal{I}^\Lambda \eta_{\Lambda\Sigma} \mathcal{I}^\Sigma - 2 \mathcal{I}_\Lambda \eta^{\Lambda\Sigma} \mathcal{I}_\Sigma = \frac{1}{2} [\mathcal{I}^{02} - \mathcal{I}^{i2} + 4\mathcal{I}_0^2 - 4\mathcal{I}_i^2] . \quad (3.6)$$

Let us then consider the case  $n = 3$ : due to Eq. (2.3)  $\mathcal{I}_a$  is proportional to  $\mathcal{I}^a$  and using the hedgehog Ansatz Eq. (2.1,2.2) we obtain

$$-g_{rr} = \frac{1}{2|X|^2} = \frac{1}{2} \left\{ \mathcal{I}^{02} + 4\mathcal{I}_0^2 - 2\mu^2 \left[ \frac{1}{g^2} + \mathcal{J}^2 \right] \mathsf{H}_\rho^2(\mu r) \right\} . \quad (3.7)$$

At the level of the metric the system behaves as if we were dealing with a  $U(1)$  field instead of an  $SU(2)$  field!

Let us then try to find a regular embedding of the 't Hooft-Polyakov monopole in the  $\overline{\mathbb{CP}}^3$  model: since the function  $\mathsf{H}_0(\mu r)$  is bound, it is enough for  $\mathcal{I}^0$  and  $\mathcal{I}_0$  to be constant as to insure that the scalars satisfy the constraint Eq. (3.4). Actually, taking them to be spherically symmetric, harmonic functions would produce scalars that violate said constraint and introduce singularities. Fixing the values of  $\mathcal{I}^0$  and  $\mathcal{I}_0$  by imposing asymptotic flatness we find

$$-g_{rr} = \frac{1}{2|X|^2} = 1 + \mu^2 \left[ \frac{1}{g^2} + \mathcal{J}^2 \right] (1 - \mathsf{H}^2(\mu r)) , \quad (3.8)$$

which means that the metric is perfectly regular and describes an object of mass

$$M = \mu \left[ \frac{1}{g^2} + \mathcal{J}^2 \right] . \quad (3.9)$$

Let us now consider the black hedgehog case. Since the function  $\mathsf{H}_\infty(\mu r)$  is singular, in order to produce scalar fields that satisfy the bound Eq. (3.4), either  $\mathcal{I}^0$  or  $\mathcal{I}_0$  has to be unfrozen, *i.e.* a non-constant harmonic function.

Choosing for simplicity

$$\mathcal{I}^0 = \mathcal{I}_\infty^0 + \frac{p^0}{r} , \quad (3.10)$$

we get

$$\begin{aligned}
-g_{rr} = \frac{1}{2|X|^2} &= \frac{1}{2} \left\{ \mathcal{I}_\infty^0 - 2\mu^2 \left[ \frac{1}{g^2} + \mathcal{J}^2 \right] \right\} \\
&+ \left\{ \mathcal{I}_\infty^0 p^0 - 2|\mu| \left[ \frac{1}{g^2} + \mathcal{J}^2 \right] \right\} \frac{1}{r} \\
&+ \frac{1}{2} \left\{ p^{02} - 2 \left[ \frac{1}{g^2} + \mathcal{J}^2 \right] \right\} \frac{1}{r^2}.
\end{aligned} \tag{3.11}$$

The first term has to be normalized to 1 to have asymptotic flatness. The coefficient of the second term is the mass and should be positive; the coefficient of the last term, iff positive, is the area of an event horizon divided by  $4\pi$ . A metric describes a regular black hole if it is asymptotically flat, has a horizon and the mass and the entropy are positive definite.

It is always possible to choose the parameters such as to obtain a regular black hole. A simple choice is

$$\mathcal{I}_\infty^0 = \sqrt{2} \sqrt{1 + \mu^2 \left[ \frac{1}{g^2} + \mathcal{J}^2 \right]}, \quad p^0 = |\mu|^{-1} \mathcal{I}_\infty^0, \tag{3.12}$$

and gives a mass and event horizon area

$$\begin{aligned}
M &= 2|\mu|^{-1}, \\
A &= 4\pi|\mu|^{-2}.
\end{aligned} \tag{3.13}$$

On the event horizon the scalars  $Z^a$  take the values

$$Z^a = \frac{\sqrt{2}}{p^0} \left( \frac{1}{g} - i\mathcal{J} \right) n^a, \tag{3.14}$$

which are independent of their asymptotic values, but not constant over the horizon. Actually, since these scalars are charged, the most we can ask for is that they are constant up to  $SO(3)$  gauge transformations (*i.e.* covariantly constant), which is the case. The scalar fields have a *covariant attractor* on the horizon and their gauge-invariant combination  $|Z|^2$  has a standard attractor.

## 4 Embedding in $\mathcal{ST}[2, n]$ models

We are now going to consider the embedding of the 't Hooft-Polyakov monopole and the black hedgehog into a more stringy model of the  $\mathcal{ST}[2, n]$  family.

Let us start by giving the symplectic section

$$\Omega = \begin{pmatrix} \mathcal{X}^\Lambda \\ \eta_{\Lambda\Sigma} \mathcal{X}^\Sigma \mathcal{S} \end{pmatrix} \quad \text{with} \quad \mathcal{X} \cdot \mathcal{X} \equiv \eta_{\Lambda\Sigma} \mathcal{X}^\Lambda \mathcal{X}^\Sigma = 0, \tag{4.1}$$

where the metric  $\eta = \text{diag}([+]^2, [-]^n)$ . In this parameterization no prepotential exists but we can do a symplectic transformation such that a prepotential exists. The Kähler potential is

$$e^{-\mathcal{K}} = -2 \Im \mathcal{S} \mathcal{X} \cdot \mathcal{X}^*. \quad (4.2)$$

The stabilisation equation was solved in Ref. [25]. Using the notation

$$p^\Lambda \equiv \Im \mathcal{X}^\Lambda, \quad q_\Lambda \equiv \eta_{\Lambda\Sigma} \Im(\mathcal{S} \mathcal{X}^\Lambda), \quad (4.3)$$

the solution takes the form

$$\mathcal{S} = \frac{p \cdot q}{p \cdot p} + i \frac{(p \cdot p q \cdot q - (p \cdot q)^2)^{1/2}}{p \cdot p}, \quad \mathcal{X} \cdot \mathcal{X}^* = p \cdot p. \quad (4.4)$$

Knowing the solution to the stabilisation equation it is straightforward to derive the metrical factor of our solutions as

$$-g_{rr} = \frac{1}{2|X|^2} = \sqrt{p \cdot p q \cdot q - (p \cdot q)^2}, \quad (4.5)$$

where we must substitute  $p^\Lambda = \mathcal{I}^\Lambda$  and  $q_\Lambda = \mathcal{I}_\Lambda$ .

Let us then restrict ourselves to the  $\mathcal{ST}[2, 3]$  model and gauge the  $SO(3)$  group. Using indices  $i, j = 1, 2$  for the first two components (which we assume correspond to ungauged directions) and taking into account Eqs. (2.3,2.1) and (2.2), the metric factor can be written in the form

$$-g_{rr} = \frac{1}{2|X|^2} = \sqrt{\mathcal{I}^i \mathcal{I}^i \mathcal{I}_j \mathcal{I}_j - (\mathcal{I}^i \mathcal{I}_i)^2 - \left(\mathcal{I}_i - \frac{g\mathcal{J}}{2} \mathcal{I}^i\right) \left(\mathcal{I}_i - \frac{g\mathcal{J}}{2} \mathcal{I}^i\right) \mathsf{H}_\rho^2(\mu r)}. \quad (4.6)$$

Again, the  $SU(2)$  fields enter effectively the metric as a  $U(1)$  field.

The 't Hooft-Polyakov monopole can be given a regular embedding in this model by taking the  $\mathcal{I}^i$  and  $\mathcal{I}_i$  to be constant. The metric function takes the form

$$-g_{rr} = \frac{1}{2|X|^2} = \sqrt{1 + 2\mu \mathsf{M} [1 - \mathsf{H}_0^2(\mu r)]}, \quad (4.7)$$

where we have normalized

$$\mathcal{I}^i \mathcal{I}^i \mathcal{I}_j \mathcal{I}_j - (\mathcal{I}^i \mathcal{I}_i)^2 - 2\mu^2 \left(\frac{1}{g} \mathcal{I}_i - \frac{\mathcal{J}}{2} \mathcal{I}^i\right) \left(\frac{1}{g} \mathcal{I}_i - \frac{\mathcal{J}}{2} \mathcal{I}^i\right) = 1. \quad (4.8)$$

The mass is given by

$$\mathsf{M} = \mu \left(\frac{1}{g} \mathcal{I}_i - \frac{\mathcal{J}}{2} \mathcal{I}^i\right) \left(\frac{1}{g} \mathcal{I}_i - \frac{\mathcal{J}}{2} \mathcal{I}^i\right), \quad (4.9)$$

and is manifestly positive if  $\mu$  is, automatically making the metric completely regular. Again, the spacetime has no event horizons.

The black hedgehog can also be given a regular embedding in this model, and requires of the introduction of a second unfrozen  $U(1)$  field. By choosing the parameters judiciously, the mass will be positive and the area of the horizon will be finite, leading to a regular black hole. The comments on covariant attractors made in the  $\overline{\mathbb{CP}}^n$  case apply to this case without any change.

## 5 Conclusions and outlook

In this paper we have given the general recipe to construct supersymmetric solutions in the timelike class of  $N = 2$  Super Einstein-Yang-Mills theories and we have shown the generic existence in  $N = 2$  Einstein-Yang-Mills theories with an  $SO(3)$  gauge group of regular, extreme, supersymmetric non-Abelian black holes (black hedgehogs) and monopoles. The monopole solutions found long ago in Refs. [4, 5] should be particular examples of this general class of monopole solutions. On the other hand, the  $SU(2) \times U(1)$  black hole solution of Ref. [26] should also belong to the class of black hedgehogs, although finding the exact correspondence is a difficult task.

We have shown that, at least in the cases considered, there is a *covariant attractor mechanism* and work on a general proof in under way.

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